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# Semi-inclusive decays $\Lambda_b \rightarrow X_c + (D_s, D_s^*)$ at $O(\alpha_s)$ including $\Lambda_b$ and $D_s^*$ polarization effects

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## Abstract

In the leading order of the  $(1/m_b)$ -expansion in HQET the dominant contribution to the semi-inclusive decays of polarized  $\Lambda_b$  baryons into the charm-strangeness mesons  $D_s$  and  $D_s^*$  is given by the partonic process  $b(\uparrow) \rightarrow c + (D_s^-, D_s^{*-})$ . Using standard values for the parameters of the process one expects a rather large branching ratio of  $\approx 8\%$  into these two channels. In the factorization approximation the semi-inclusive decay of a polarized  $\Lambda_b$  is governed by three unpolarized and four polarized structure functions for which we determine the nonperturbative  $O(1/m_b^2)$  corrections and the  $O(\alpha_s)$  radiative corrections. We find that the perturbative and nonperturbative corrections amount to  $\approx 10\%$  and  $\approx 3\%$ , respectively. The seven structure functions can be measured through an analysis of the joint decay distributions of the process involving the polarization of the  $\Lambda_b$  and the decays  $D_s^{*-} \rightarrow D_s^- + \gamma$  and  $D_s^{*-} \rightarrow D_s^- + \pi^0$  for which we provide explicit forms. We also provide numerical results for the Cabibbo suppressed semi-inclusive decays  $\Lambda_b \rightarrow X_u + (D_s, D_s^*)$ .

# 1 Introduction

In the leading order of the  $(1/m_b)$ -expansion in HQET the semi-inclusive decay  $\Lambda_b \rightarrow X_c + (D_s^-, D_s^{*-})$  is dominated by the partonic process  $b \rightarrow c + (D_s^-, D_s^{*-})$ . The basic assumption is that factorization holds for the nonleptonic decay process  $\Lambda_b \rightarrow X_c + (D_s^-, D_s^{*-})$ . One can then factorize the semi-inclusive decay into a current-induced  $\Lambda_b \rightarrow X_c$  transition and a current-induced vacuum one-meson transition. The leading order  $1/m_Q$  contribution to the  $\Lambda_b \rightarrow X_c$  transition is given by the partonic  $b \rightarrow c$  transition. There are two types of corrections to the leading order result. First there are the nonperturbative corrections which set in at  $O(1/m_b^2)$  in the heavy mass expansion. They can be estimated using the methods of the operator product expansion in HQET. Second there are also the perturbative  $O(\alpha_s)$  corrections which can be calculated using standard techniques. From a previous calculation of the corresponding decays in the mesonic sector  $\bar{B}^0 \rightarrow X_c + (D_s^-, D_s^{*-})$  one expects perturbative and nonperturbative corrections of  $\approx 10\%$  [1, 2] and  $\approx 1\%$  [2], respectively.

When the  $\Lambda_b$  is unpolarized, the decay  $\Lambda_b \rightarrow X_c + (D_s^-, D_s^{*-})$  is quite similar to the corresponding mesonic decay  $\bar{B}^0 \rightarrow X_c + (D_s^-, D_s^{*-})$  [1, 2]. In fact, to leading order in the  $1/m_b$  expansion and to any order in the perturbative QCD corrections the two semi-inclusive decays are identical to one another. However, when the  $\Lambda_b$  is polarized, there are four additional polarized structure functions that enter the decay analysis. One can thus probe four more structure functions in the semi-inclusive decay of a polarized  $\Lambda_b$  than it is possible in the corresponding  $B$ -meson decay. Polarized  $b$ -quarks and thereby polarized  $\Lambda_b$  baryons arise quite naturally in weak decays such as  $Z \rightarrow b\bar{b}$  and  $t \rightarrow Wb$ . When the polarized  $b$ -quark fragments into a  $\Lambda_b$  baryon,  $\approx 70\%$  of its polarization is retained [3, 4].

We mention that large samples of  $\Lambda_b$ 's are expected to be produced at the currently running  $p\bar{p}$  collider Tevatron 2. In fact the first few  $\Lambda_b$ 's have been reconstructed by the CDF collaboration using the superior tracking capacity of their new silicon vertex trigger [5].

## 2 Angular decay distributions

In the factorization approximation the semi-inclusive decays of polarized  $\Lambda_b$  baryons  $\Lambda_b(\uparrow) \rightarrow X_c + (D_s^-, D_s^{*-})$  are governed by altogether seven structure functions which can be measured by an angular analysis of the decay process. We mention that there are two addi-

tional parity-violating structure functions in the decays  $\Lambda_b(\uparrow) \rightarrow X_c + D_s^{*-}$  which, however, cannot be measured since the dominating decays of the  $D_s^{*-}$  are parity-conserving.

Five of the structure functions describe the semi-inclusive decay  $\Lambda_b \rightarrow X_c + D_s^{*-}$  into vector mesons followed by their subsequent decay into  $D_s^{*-} \rightarrow D_s^- + \gamma$  and  $D_s^{*-} \rightarrow D_s^- + \pi^0$ . The branching ratios of the  $D_s^{*-}$  into these two principal channels are given by  $(94.2 \pm 2.5)\%$  and  $(5.8 \pm 2.5)\%$  [6], respectively.

The angular decay distribution of the semi-inclusive polarized  $\Lambda_b$  decays can be obtained from the master formula (see e.g. [7])

$$W(\theta_P, \theta, \phi) \propto \sum_{\lambda_{D_s^*} - \lambda'_{D_s^*} = \lambda_{\Lambda_b} - \lambda'_{\Lambda_b}} e^{i(\lambda_{D_s^*} - \lambda'_{D_s^*})\phi} d_{\lambda_{D_s^*}^* m}^1(\theta) d_{\lambda'_{D_s^*}^* m}^1(\theta) H_{\lambda_{D_s^*}^* \lambda'_{D_s^*}^*}^{\lambda_{\Lambda_b} \lambda'_{\Lambda_b}} \rho_{\lambda_{\Lambda_b} \lambda'_{\Lambda_b}}(\theta_P), \quad (1)$$

where  $\rho_{\lambda_{\Lambda_b} \lambda'_{\Lambda_b}}(\theta_P)$  is the density matrix of the  $\Lambda_b$  which reads

$$\rho_{\lambda_{\Lambda_b} \lambda'_{\Lambda_b}}(\theta_P) = \frac{1}{2} \begin{pmatrix} 1 + P \cos \theta_P & P \sin \theta_P \\ P \sin \theta_P & 1 - P \cos \theta_P \end{pmatrix}. \quad (2)$$

$P$  is the magnitude of the polarization of the  $\Lambda_b$ . The  $H_{\lambda_{D_s^*}^* \lambda'_{D_s^*}^*}^{\lambda_{\Lambda_b} \lambda'_{\Lambda_b}}$  are the helicity components of the hadronic tensor  $H_{\mu\nu}(s_{\Lambda_b})$  describing the semi-inclusive decay. The sum in Eq. (1) extends over all values of  $\lambda_{D_s^*}^*$ ,  $\lambda'_{D_s^*}^*$ ,  $\lambda_{\Lambda_b}$  and  $\lambda'_{\Lambda_b}$  compatible with the constraint  $\lambda_W - \lambda'_W = \lambda_{\Lambda_b} - \lambda'_{\Lambda_b}$  (the spin degrees of freedom of  $X_c$  are being summed over). The polar angles  $\theta_P$ ,  $\theta$  and the azimuthal angle  $\phi$  are defined in Fig. 1. Because of angular momentum conservation, the second lower index in the small Wigner  $d(\theta)$ -function  $d_{\lambda_{D_s^*}^* m}^1(\theta)$  runs over  $m = \pm 1$  for the decay  $D_s^{*-} \rightarrow D_s^- + \gamma$  and over  $m = 0$  for the decay  $D_s^{*-} \rightarrow D_s^- + \pi^0$ . One thus obtains the angular decay distributions

$$\begin{aligned} \frac{d\Gamma_{\Lambda_b^{(\uparrow)} \rightarrow X_c + D_s^{*-} (\rightarrow D_s^- + \gamma)}}{d\cos \theta_P d\cos \theta d\phi} &= \frac{1}{4\pi} \text{BR}(D_s^{*-} \rightarrow D_s^- + \gamma) \left\{ \frac{3}{8} (1 + \cos^2 \theta) (\Gamma_U + \Gamma_{UP} P \cos \theta_P) \right. \\ &\quad \left. + \frac{3}{4} \sin^2 \theta (\Gamma_L + \Gamma_{LP} P \cos \theta_P) + \frac{3}{4} \sqrt{2} P \sin \theta_P \sin 2\theta \cos \phi \Gamma_{IP} \right\} \end{aligned} \quad (3)$$

and

$$\begin{aligned} \frac{d\Gamma_{\Lambda_b^{(\uparrow)} \rightarrow X_c + D_s^{*-} (\rightarrow D_s^- + \pi^0)}}{d\cos \theta_P d\cos \theta d\phi} &= \frac{1}{4\pi} \text{BR}(D_s^{*-} \rightarrow D_s^- + \pi^0) \left\{ \frac{3}{4} \sin^2 \theta (\Gamma_U + \Gamma_{UP} P \cos \theta_P) \right. \\ &\quad \left. + \frac{3}{2} \cos^2 \theta (\Gamma_L + \Gamma_{LP} P \cos \theta_P) - \frac{3}{2} \sqrt{2} P \sin \theta_P \sin 2\theta \cos \phi \Gamma_{IP} \right\}. \end{aligned} \quad (4)$$

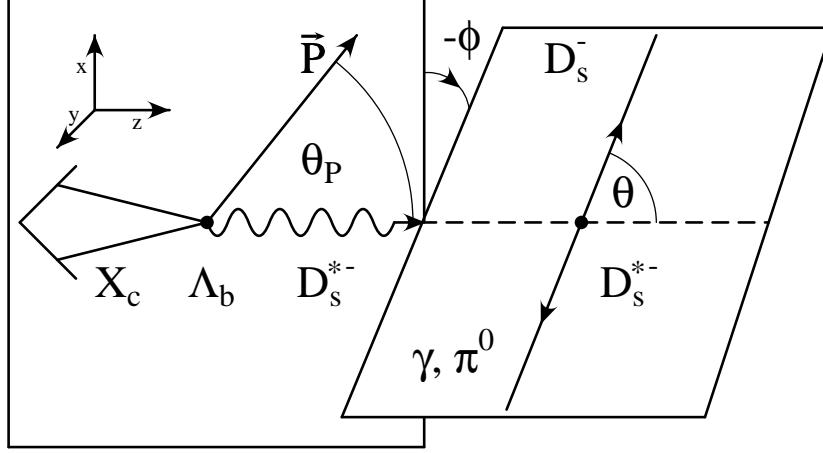


Figure 1: Definition of polar angles  $\theta_P$  and  $\theta$ , and the azimuthal angle  $\phi$  in the semi-inclusive decay  $\Lambda_b(\uparrow) \rightarrow X_c + D_s^* (\rightarrow D_s^- + \gamma \text{ or } \pi^0)$ .  $\vec{P}$  is the polarization vector of the  $\Lambda_b$ . The polar angle  $\theta$  is defined in the  $D_s^{*-}$  rest frame relative to the direction of the  $D_s^{*-}$  in the  $\Lambda_b$  rest frame.

Two-fold or single angle decay distributions can be obtained from Eqs. (3,4) by further integration. For example, the single angle dependence on  $\cos \theta_P$  for both cases is given by

$$\begin{aligned} \frac{d\Gamma_{\Lambda_b(\uparrow) \rightarrow X_c + D_s^{*-}}}{d\cos \theta_P} &= \frac{1}{2}(\Gamma_{U+L} + \Gamma_{(U+L)^P} P \cos \theta_P) \\ &= \frac{1}{2}\Gamma_{U+L}(1 + \alpha_P(D_s^*) P \cos \theta_P), \end{aligned} \quad (5)$$

where we have defined an asymmetry parameter  $\alpha_P(D_s^*) = \Gamma_{(U+L)^P}/\Gamma_{U+L}$ .

The transverse/longitudinal composition of the vector meson  $D_s^{*-}$  can be best determined by analyzing the  $\cos \theta$ -dependence of the decay distributions after integrating over  $\cos \theta_P$  and  $\phi$ . Note that the  $\cos \theta$ -dependence is different in the two decay modes.

The decay distribution for  $\Lambda_b(\uparrow) \rightarrow X_c + D_s^-$  can be obtained from the same master formula (1) with the appropriate substitutions  $\lambda_{D_s^*} \rightarrow \lambda_{D_s} = 0$  and  $d^1 \rightarrow d^0 = 1$ . The helicity of the  $\lambda_{D_s}$  will be denoted by the symbol “S” for “scalar”. One has

$$\begin{aligned} \frac{d\Gamma_{\Lambda_b(\uparrow) \rightarrow X_c + D_s^-}}{d\cos \theta_P} &= \frac{1}{2}(\Gamma_S + \Gamma_{S^P} P \cos \theta_P) \\ &= \frac{1}{2}\Gamma_S(1 + \alpha_P(D_s) P \cos \theta_P), \end{aligned} \quad (6)$$

where we have again defined an asymmetry parameter  $\alpha_P(D_s) = \Gamma_{S^P}/\Gamma_S$ .

The angular coefficients  $\Gamma_i$  ( $i = S, S^P, U, L, U^P, L^P, I^P$ ) appearing in the decay distributions are partial helicity rates defined by

$$\Gamma_i = \frac{G_F^2}{8\pi} |V_{bc} V_{cs}^*|^2 f_{D_s^{(*)}}^2 m_b^2 p_{D_s^{(*)}} a_1^2 H_i, \quad (7)$$

where the helicity structure functions  $H_i$  are linear combinations of the helicity components. They read

$$\begin{aligned} H_S &= H_{SS}^{++} + H_{SS}^{--}, \\ H_U &= H_{++}^{++} + H_{++}^{--} + H_{--}^{++} + H_{--}^{--}, \\ H_L &= H_{00}^{++} + H_{00}^{--}, \\ H_{S^P} &= H_{SS}^{++} - H_{SS}^{--}, \\ H_{U^P} &= H_{++}^{++} - H_{++}^{--} + H_{--}^{++} - H_{--}^{--}, \\ H_{L^P} &= H_{00}^{++} - H_{00}^{--}, \\ H_{I^P} &= \frac{1}{4}(H_{+0}^{+-} + H_{0+}^{-+} - H_{-0}^{+-} - H_{0-}^{-+}) = \frac{1}{2}(H_{+0}^{+-} - H_{-0}^{-+}), \end{aligned} \quad (8)$$

where, for the ease of writing, we have omitted factors of  $1/2$  in the upper indices standing for the helicities of the  $\Lambda_b$ . The remaining quantities appearing in (7) are defined in Sec. 3.

When the  $\Lambda_b$  is unpolarized ( $P = 0$ ), or when one integrates over the angles  $\theta_P$  and  $\phi$  that describe the orientation of the polarization vector of the  $\Lambda_b$ , one remains with the contributions of the three structure functions  $H_U$ ,  $H_L$  and  $H_S$  in the decay distributions. In this way one recovers the decay distributions for the corresponding semi-inclusive decays of  $B$  mesons into  $D_s$  and  $D_s^*$  treated in [2].

### 3 Born term rates

As explained in Sec. 2, the decay  $\Lambda_b \rightarrow X_c + (D_s^-, D_s^{*-})$  involves seven structure functions which can be resolved by an angular analysis of the decay products. We begin by writing down the leading order Born term contributions given by the quark level transition  $b \rightarrow c + (D_s^-, D_s^{*-})$  (see Fig. 2a). For the partial helicity rates one obtains

$$\Gamma_i^{\text{Born}}(b^{(\uparrow)} \rightarrow c + D_s^{(*)-}) = \frac{G_F^2}{8\pi} |V_{bc} V_{cs}^*|^2 f_{D_s^{(*)}}^2 m_b^2 p_{D_s^{(*)}} a_1^2 B_i, \quad (9)$$

where

$$\begin{aligned}
B_S = B_L &= (1 - y^2)^2 - x^2(1 + y^2), \\
B_U &= 2x^2(1 - x^2 + y^2), \\
B_{U+L} &= (1 - y^2)^2 + x^2(1 + y^2 - 2x^2), \\
B_{S^P} = B_{L^P} &= \sqrt{\lambda}(1 - y^2), \\
B_{U^P} &= -2x^2\sqrt{\lambda}, \\
B_{(U+L)^P} &= \sqrt{\lambda}(1 - y^2 - 2x^2), \\
B_{I^P} &= -\frac{1}{\sqrt{2}}x\sqrt{\lambda},
\end{aligned} \tag{10}$$

and where  $x = m_{D_s^{(*)}}/m_b$  and  $y = m_c/m_b$ . The kinematical factor  $\lambda$  is defined by  $\lambda = 1 + x^4 + y^4 - 2(x^2 + y^2 + x^2y^2)$  such that  $p_{D_s^{(*)}} = \frac{1}{2}m_b\lambda^{1/2}$ . In Eq. (9),  $f_{D_s}$  and  $f_{D_s^*}$  denote the pseudoscalar and vector meson coupling constants defined by  $\langle D_s^- | A^\mu | 0 \rangle = if_{D_s} p_{D_s}^\mu$  and  $\langle D_s^{*-} | V^\mu | 0 \rangle = f_{D_s^*} m_{D_s^*} \epsilon^{*\mu}$ , respectively. The Kobayashi–Maskawa matrix element is denoted by  $V_{q_1 q_2}$ , and the  $p_{D_s}$  and  $p_{D_s^*}$  are the magnitude of the three-momenta of the  $D_s$  and  $D_s^*$  in the  $b$  rest system. The parameter  $a_1$  is related to the Wilson coefficients of the renormalized current-current interaction and is obtained from a combined fit of several decay modes ( $|a_1| = 1.00 \pm 0.06$ ) [1]. Note that the structural similarity of the unpolarized and polarized rate formulae for the decay into  $D_s$  and into the longitudinal  $D_s^*$  is an accident of the Born term calculation and does not persist e.g. at higher orders of  $\alpha_s$ , or for the nonperturbative contributions to the unpolarized longitudinal rate into  $D_s^*$  to be written down later on.

$\Gamma_S^{\text{Born}}$  and  $\Gamma_{U+L}^{\text{Born}}$  determine the total  $\Lambda_b \rightarrow X_c + D_s^-$  and  $\Lambda_b \rightarrow X_c + D_s^{*-}$  rates at the Born term level, respectively. Using  $f_{D_s} = 230$  MeV and  $f_{D_s^*} = 280$  MeV as in [1],  $\tau_{\Lambda_b} = 1.23$  ps,  $V_{bc} = 0.04$ ,  $V_{cs} = 0.974$  and the central value for  $a_1$ , one arrives at

$$\text{BR}_{\Lambda_b \rightarrow X_c + D_s^-} \cong 2.5\%, \quad \text{BR}_{\Lambda_b \rightarrow X_c + D_s^{*-}} \cong 5.2\%. \tag{11}$$

While the semi-inclusive  $\Lambda_b \rightarrow X_c + (D_s^-, D_s^{*-})$  rates have not been measured yet, a comparison of the Born term prediction with data on the corresponding mesonic decay  $\bar{B}^0 \rightarrow X_c + (D_s^-, D_s^{*-})$  is meaningful because the Born level predictions for both processes are identical. Allowing for the factor  $\tau_{\Lambda_b}/\tau_B \approx 0.77$  and summing up the  $D_s$  and  $D_s^*$

modes, one arrives at a branching ratio of 10% which is consistent with the measured value  $\text{BR}(B \rightarrow X + D_s^\pm) = (10.0 \pm 2.5)\%$  [6] if one assumes that the above two rates saturate the semi-inclusive rate into  $D_s^\pm$ .

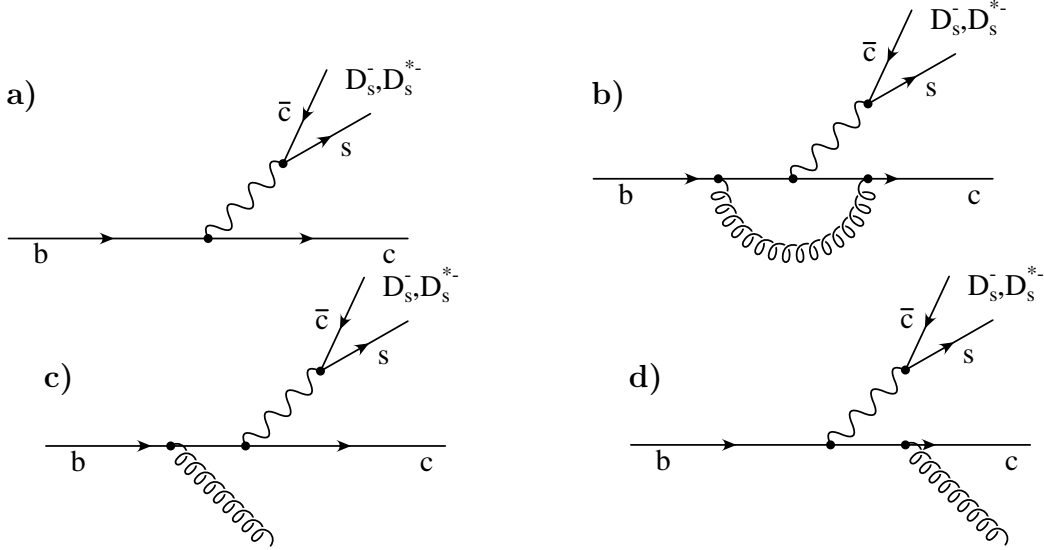


Figure 2: Leading order Born term contribution (a) and  $O(\alpha_s)$  contributions (b, c, d) to  $b \rightarrow c + (D_s^-, D_s^{*-})$ .

## 4 $O(\alpha_s)$ radiative corrections

Next we turn to the  $O(\alpha_s)$  radiative corrections. As is evident from Fig. 2, the radiative gluon corrections connect only to the  $b$  and  $c$  legs of the parton decay process  $b \rightarrow c + (D_s, D_s^*)$  because of the conservation of colour (see Fig. 2b, 2c and 2d). The radiative corrections for the seven structure functions are thus identical to the corresponding radiative corrections calculated in [7] where the process  $t \rightarrow W^+ + b$  was considered (including the scalar case) keeping  $m_b \neq 0$ <sup>1</sup>.

For the  $O(\alpha_s)$  radiative corrections one has to calculate the square of the tree-graph amplitudes Figs. 2c and 2d, and the one-loop contribution Fig. 2b. We concentrate on the tree-graph contribution given by the squares of the tree-graph amplitudes Fig. 2b and 2c which will be denoted by  $\mathcal{H}^{\mu\nu}(tree)$ . The  $b \rightarrow c$  hadron tensor can be obtained from the

<sup>1</sup>In the unpolarized case the total  $O(\alpha_s)$  correction to the spin 1 piece of the weak current keeping both quark masses finite had been calculated before in [8, 9, 10, 11]. The  $O(\alpha_s)$  corrections to the (unpolarized) spin 0 piece of the weak current can also be deduced from the calculations of [8, 12].

corresponding  $t \rightarrow b$  hadron tensor given in [7] by the replacements  $p_t \rightarrow p_b$  and  $p_b \rightarrow p_c$ <sup>2</sup>. One obtains

$$\begin{aligned}
\mathcal{H}^{\mu\nu}(tree) = & 4\pi\alpha_s C_F \frac{8}{(k \cdot p_b)(k \cdot p_c)} \left\{ -\frac{k \cdot p_b}{k \cdot p_c} \left[ m_c^2 \left( k^\mu \bar{p}_b^\nu + k^\nu \bar{p}_b^\mu - k \cdot \bar{p}_b g^{\mu\nu} \right) + \right. \\
& + i \left( \epsilon^{\alpha\beta\mu\nu} (p_c - k) \cdot \bar{p}_b - \epsilon^{\alpha\beta\gamma\nu} (p_c - k)^\mu \bar{p}_{b,\gamma} + \epsilon^{\alpha\beta\gamma\mu} (p_c - k)^\nu \bar{p}_{b,\gamma} \right) k_\alpha p_{c,\beta} \Big] + \\
& + \frac{k \cdot p_c}{k \cdot p_b} \left[ (\bar{p}_b \cdot p_b) \left( k^\mu p_c^\nu + k^\nu p_c^\mu - k \cdot p_c g^{\mu\nu} - i \epsilon^{\alpha\beta\mu\nu} k_\alpha p_{c,\beta} \right) + \right. \\
& - (\bar{p}_b \cdot k) \left( (p_b - k)^\mu p_b^\nu + (p_b - k)^\nu p_b^\mu - (p_b - k) \cdot p_c g^{\mu\nu} - i \epsilon^{\alpha\beta\mu\nu} (p_b - k)_\alpha p_{c,\beta} \right) \Big] + \\
& - (\bar{p}_b \cdot p_c) \left( k^\mu p_c^\nu + k^\nu p_c^\mu - k \cdot p_c g^{\mu\nu} - i \epsilon^{\alpha\beta\mu\nu} k_\alpha p_{c,\beta} \right) + (p_b \cdot p_c) \left( k^\mu \bar{p}_b^\nu + k^\nu \bar{p}_b^\mu - k \cdot \bar{p}_b g^{\mu\nu} \right) + \\
& - (k \cdot p_c) \left( p_b^\mu \bar{p}_b^\nu + p_b^\nu \bar{p}_b^\mu - p_b \cdot \bar{p}_b g^{\mu\nu} \right) + (k \cdot p_b) \left( (p_c + k)^\mu \bar{p}_b^\nu + (p_c + k)^\nu \bar{p}_b^\mu - (p_c + k) \cdot \bar{p}_b g^{\mu\nu} \right) + \\
& + (k \cdot \bar{p}_b) \left( 2p_c^\mu p_c^\nu - p_c \cdot p_c g^{\mu\nu} \right) + i \left( \epsilon^{\alpha\beta\mu\nu} (k \cdot \bar{p}_b) + \epsilon^{\alpha\beta\gamma\mu} k^\nu \bar{p}_{b,\gamma} - \epsilon^{\alpha\beta\gamma\nu} k^\mu \bar{p}_{b,\gamma} \right) p_{c,\alpha} p_{b,\beta} + \\
& \left. + i \left( \epsilon^{\alpha\beta\mu\nu} (p_b \cdot \bar{p}_b) + \epsilon^{\alpha\beta\gamma\mu} p_b^\nu \bar{p}_{b,\gamma} - \epsilon^{\alpha\beta\gamma\nu} p_b^\mu \bar{p}_{b,\gamma} \right) k_\alpha p_{c,\beta} \right\} + B^{\mu\nu} \cdot \Delta_{SGF}
\end{aligned} \tag{12}$$

$$\Delta_{SGF} := -4\pi\alpha_s C_F \left( \frac{m_c^2}{(k \cdot p_c)^2} + \frac{m_b^2}{(k \cdot p_b)^2} - 2 \frac{p_c \cdot p_b}{(k \cdot p_c)(k \cdot p_b)} \right) \tag{13}$$

where  $k$  is the 4-momentum of the emitted gluon. The polarization of the bottom quark is taken into account by introducing the short-hand notation  $\bar{p}_b = p_b - m_b s_b$ . We have found it convenient to split the tree-graph hadron tensor into an infrared (IR) finite piece and an IR divergent piece given by the usual soft-gluon factor  $\Delta_{SGF}$  multiplied by the Born term tensor  $B^{\mu\nu}$

$$B^{\mu\nu} = 2(\bar{p}_b^\nu p_c^\mu + \bar{p}_b^\mu p_c^\nu - g^{\mu\nu} \bar{p}_b \cdot p_c + i \epsilon^{\mu\nu\alpha\beta} p_{c,\alpha} \bar{p}_{b,\beta}). \tag{14}$$

In this way the IR singularity is isolated in the universal function  $\Delta_{SGF}$  which can be integrated by introducing a gluon mass regulator to regularize the IR singularity. The ensuing logarithmic gluon mass singularity is cancelled by the corresponding gluon mass singularity occurring in the loop contribution (see e.g.[7]).

The phase-space integration of the IR convergent piece can be done without a gluon mass regulator. One first projects the convergent piece of the tree-graph tensor (12) onto the seven helicity structure functions Eq.(8) and then does the phase-space integration in the sequential order (i)  $k_0$  (gluon energy) (ii)  $q_0$  (energy of the off-shell  $W^-$ ).

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<sup>2</sup>We take the opportunity to correct two sign typos in the corresponding  $t \rightarrow b$  expression in [7]

The final  $O(\alpha_s)$  answer including the one-loop contribution can be written in a very compact way by introducing combinations of dilogarithmic functions  $\mathcal{A}$  and  $\mathcal{N}_{0,\dots,4}$ . The contribution denoted by  $\mathcal{A}$  is part of the finite remainder of the Born term type one-loop contribution plus the soft gluon contribution. The combinations  $\mathcal{N}_{0,\dots,4}$  appear when integrating those helicity structure functions  $H_{SP}, H_U, H_{UP}, H_{LP}, H_{LP}$  and  $H_{IP}$  that are not associated with the total rate. All these functions are defined after Eq.(24). We mention that we have now been able to present our results on the radiatively corrected structure functions in a much more compact form than thought possible when we wrote up [7].

We shall present our  $O(\alpha_s)$  results in a form where the respective Born terms  $\Gamma_i^{(0)}$  are factored out from the  $O(\alpha_s)$  result. Including the Born term and the nonperturbative  $O(1/m_b^2)$  contributions to be discussed in Sec. 4 we write  $\hat{\Gamma}_i := \Gamma_i/\Gamma_S^{\text{Born}}$  and  $\hat{\Gamma}_i^{\text{Born}} := \Gamma_i^{\text{Born}}/\Gamma_S^{\text{Born}}$  for  $i = S, S^P$ , and  $\hat{\Gamma}_i := \Gamma_i/\Gamma_{U+L}^{\text{Born}}$  and  $\hat{\Gamma}_i^{\text{Born}} := \Gamma_i^{\text{Born}}/\Gamma_{U+L}^{\text{Born}}$  for  $i = U, L, U + L, U^P, L^P, U^P + L^P, I^P$ . One has

$$\hat{\Gamma}_i = \hat{\Gamma}_i^{\text{Born}} \left( 1 + C_F \frac{\alpha_s}{4\pi} \tilde{\Gamma}_i + a_i^K K_b + a_i^\epsilon \epsilon_b \right). \quad (15)$$

$K_b$  is the expectation value of the kinetic energy of the heavy quark in the  $\Lambda_b$  baryon and  $\epsilon_b$  parametrizes the spin-dependent contribution of the heavy quark in the  $\Lambda_b$  baryon [13].

To begin with, we list the reduced  $O(\alpha_s)$  rates  $\tilde{\Gamma}_i$ . For the reduced unpolarized and polarized scalar spin 0 rates  $\tilde{\Gamma}_S$  and  $\tilde{\Gamma}_{S^P}$  we obtain

$$\begin{aligned} \tilde{\Gamma}_S = & \mathcal{A} + \frac{1}{\sqrt{\lambda}} \left[ \lambda + x^2 (1 - x^2 + y^2) \right]^{-1} \\ & \times \left\{ \frac{2}{x^2} \left[ (1 - x^2) (2 - x^2) - (6 + 4x^2 + 5x^4) y^2 \right. \right. \\ & \quad \left. \left. + (6 + 7x^2) y^4 - 2y^6 \right] \sqrt{\lambda} \ln(y) \right. \\ & \quad \left. + 8 \left[ 1 - x^2 - (2 + x^2) y^2 + y^4 \right] \sqrt{\lambda} \ln \left( \frac{xy}{\lambda} \right) \right. \\ & \quad \left. - \frac{1}{x^2} \left[ (1 - x^2)^2 (2 + 3x^2) - (8 - 3x^2 + 4x^4 - 3x^6) y^2 \right. \right. \\ & \quad \left. \left. + 3(4 + 5x^2) y^4 - (8 + 5x^2) y^6 + 2y^8 \right] \ln(w_1) \right. \\ & \quad \left. + 8(1 - y^2) \left[ 1 - x^2 - (2 + x^2) y^2 + y^4 \right] \ln(\eta) \right. \\ & \quad \left. + 3\sqrt{\lambda} \left[ 3(1 - x^2) - (10 + 3x^2) y^2 + 3y^4 \right] \right\} \quad (16) \end{aligned}$$

$$\begin{aligned}
\tilde{\Gamma}_{S^P} = & \mathcal{A} + \frac{1}{\lambda} \left\{ 4 \left[ 2 + x^4 - (3 + 2x^2) y^2 + y^4 \right] \mathcal{N}_0 + 4 \sqrt{\lambda} (1 - x^2 + y^2) \mathcal{N}_4 \right. \\
& + \frac{2}{x^2} \frac{1}{1 - y^2} \left[ 2 - x^2 - (4 + 5x^2) y^2 + 2y^4 \right] \lambda \ln(y) + 8 \lambda \ln \left( \frac{xy}{\lambda} \right) \\
& - \frac{\sqrt{\lambda}}{x^2} \left[ 2 - 9x^2 + x^4 - (4 + 3x^2) y^2 + 2y^4 \right] \ln(w_1) + 8 \lambda \ln \left[ \frac{(1+x)^2 - y^2}{x} \right] \\
& + 4 \left[ (1 - x^2) (5 - 2x^2) + 2 (2 - x^2) y^2 \right] \ln \left( \frac{1-x}{y} \right) \\
& \left. - \left[ (1-x)^2 - y^2 \right] (11 - 6x - 7x^2 + 7y^2) \right\} \quad (17)
\end{aligned}$$

The other variables and functions appearing in Eqs. (16) and (17) are explained at the end of this section. For the five unpolarized and polarized reduced spin 1 rates  $\tilde{\Gamma}_i$  ( $i = U, L, U^P, L^P, I^P$ ) we obtain

$$\begin{aligned}
\tilde{\Gamma}_U = & \mathcal{A} + \frac{1}{\sqrt{\lambda}} \frac{1}{1 - x^2 + y^2} \left\{ - 4 (7 + x^2 - y^2) \mathcal{N}_1 \right. \\
& - \frac{2}{x} \left[ (1-x)^2 - y^2 \right] \left[ (1-x)(5+x) + y^2 \right] \mathcal{N}_2 \\
& - \frac{2}{x} \left[ (1+x)^2 - y^2 \right] \left[ (1+x)(5-x) + y^2 \right] \mathcal{N}_3 \\
& - \frac{2}{x^2} (1 - x^2 + y^2) (1 - 2x^2 - y^2) \sqrt{\lambda} \ln(y) + 8 (1 - x^2 + y^2) \sqrt{\lambda} \ln \left( \frac{xy}{\lambda} \right) \\
& + \frac{1}{x^2} \left[ (1 - x^2)^2 (1 - 6x^2) \right. \\
& \quad \left. - (1 + 4x^2 - 3x^4) y^2 - (1 + 2x^2) y^4 + y^6 \right] \ln(w_1) \\
& \left. + 4 \left[ 7 + 3x^2 - (4 - 5x^2) y^2 - 3y^4 \right] \ln(\eta) - \sqrt{\lambda} (19 + x^2 - 5y^2) \right\} \quad (18)
\end{aligned}$$

$$\begin{aligned}
\tilde{\Gamma}_L = & \mathcal{A} + \frac{1}{\sqrt{\lambda}} \left[ \lambda + x^2 (1 - x^2 + y^2) \right]^{-1} \left\{ 8 x^2 (7 + x^2 - y^2) \mathcal{N}_1 \right. \\
& + 4 x \left[ (1-x)^2 - y^2 \right] \left[ (1-x)(5+x) + y^2 \right] \mathcal{N}_2 \\
& + 4 x \left[ (1+x)^2 - y^2 \right] \left[ (1+x)(5-x) + y^2 \right] \mathcal{N}_3 \\
& + 2 \left[ 1 - x^2 - (4 + 3x^2) y^2 + 3y^4 \right] \sqrt{\lambda} \ln(y) \\
& \left. + 8 \left[ 1 - x^2 - (2 + x^2) y^2 + y^4 \right] \sqrt{\lambda} \ln \left( \frac{xy}{\lambda} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& - \left[ 5 (1 - x^2)^2 - (3 + 20 x^2 - x^4) y^2 + (9 - 2 x^2) y^4 + y^6 \right] \ln(w_1) \\
& + 8 (1 + x^2 - y^2) \left[ 1 - 7 x^2 - (2 + x^2) y^2 + y^4 \right] \ln(\eta) \\
& + \sqrt{\lambda} \left[ 5 + 47 x^2 - 4 x^4 - (22 + x^2) y^2 + 5 y^4 \right] \Big\} \tag{19}
\end{aligned}$$

$$\begin{aligned}
\tilde{\Gamma}_{U+L} = & \mathcal{A} + \frac{1}{\sqrt{\lambda}} \left[ \lambda + 3 x^2 (1 - x^2 + y^2) \right]^{-1} \\
& \times \left\{ - 2 \left[ (1 - x^2) (1 - 4 x^2) + (4 + x^2) y^2 - 5 y^4 \right] \sqrt{\lambda} \ln(y) \right. \\
& + 8 \left[ (1 - x^2) (1 + 2 x^2) - (2 - x^2) y^2 + y^4 \right] \sqrt{\lambda} \ln \left( \frac{x y}{\lambda} \right) \\
& - \left[ 3 (1 - x^2)^2 (1 + 4 x^2) \right. \\
& \quad \left. - (1 + 12 x^2 + 5 x^4) y^2 + (11 + 2 x^2) y^4 - y^6 \right] \ln(w_1) \\
& + 8 (1 - y^2) \left[ 1 + x^2 - 4 x^4 - (2 - x^2) y^2 + y^4 \right] \ln(\eta) \\
& \left. + \sqrt{\lambda} \left[ 5 + 9 x^2 - 6 x^4 - (22 - 9 x^2) y^2 + 5 y^4 \right] \right\} \tag{20}
\end{aligned}$$

$$\begin{aligned}
\tilde{\Gamma}_{UP} = & \mathcal{A} + \frac{1}{\lambda} \left\{ 4 \left[ 11 + 3 x^2 + x^4 - 2 (3 + x^2) y^2 + y^4 \right] \mathcal{N}_0 + 4 \sqrt{\lambda} (1 - x^2 + y^2) \mathcal{N}_4 \right. \\
& - \frac{2}{x^2} (1 - 2 x^2 - y^2) \lambda \ln(y) + 8 \lambda \ln \left( \frac{x y}{\lambda} \right) \\
& + \frac{\sqrt{\lambda}}{x^2} \left[ 7 + 21 x^2 + 2 x^4 - (8 + 3 x^2) y^2 + y^4 \right] \ln(w_1) \\
& + 8 \lambda \ln \left[ \frac{(1 + x)^2 - y^2}{x} \right] \\
& + \frac{4}{x^2} \left[ (1 - x^2) (3 + 14 x^2 - 2 x^4) \right. \\
& \quad \left. - (6 - 7 x^2 - x^4) y^2 + (3 - x^2) y^4 \right] \ln \left( \frac{1 - x}{y} \right) \\
& \left. + \frac{1}{x} \left[ (1 - x)^2 - y^2 \right] \left[ 12 - 55 x + 6 x^2 - x^3 - 3 (4 + x) y^2 \right] \right\} \tag{21}
\end{aligned}$$

$$\tilde{\Gamma}_{LP} = \mathcal{A} + \frac{1}{\lambda} \frac{1}{1 - y^2}$$

$$\begin{aligned}
& \times \left\{ 4 \left[ 2 + 22x^2 + 11x^4 - (5 + 12x^2 + x^4)y^2 + 2(2 + x^2)y^4 - y^6 \right] \mathcal{N}_0 \right. \\
& + 4(1 - y^2)\sqrt{\lambda}(1 - x^2 + y^2)\mathcal{N}_4 \\
& + 2(1 - 3y^2)\lambda \ln(y) + 8(1 - y^2)\lambda \ln\left(\frac{xy}{\lambda}\right) \\
& + \sqrt{\lambda} \left[ 17 + 53x^2 - (18 + x^2)y^2 + y^4 \right] \ln(w_1) \\
& + 8\lambda(1 - y^2) \ln\left[\frac{(1+x)^2 - y^2}{x}\right] \\
& + 4 \left[ (1 - x^2)(11 + 24x^2) - (13 - 15x^2)y^2 + 2y^4 \right] \ln\left(\frac{1-x}{y}\right) \\
& - \left[ (1-x)^2 - y^2 \right] \\
& \left. \times \left[ 15 - 22x + 105x^2 - 24x^3 + 4x^4 - (12 - 22x + x^2)y^2 - 3y^4 \right] \right\}
\end{aligned} \tag{22}$$

$$\begin{aligned}
\tilde{\Gamma}_{U^P+L^P} = & \mathcal{A} + \frac{1}{\lambda} \frac{1}{1 - 2x^2 - y^2} \left\{ 4 \left[ 2 + 5x^4 - 2x^6 - (5 - 3x^4)y^2 + 4y^4 - y^6 \right] \mathcal{N}_0 \right. \\
& + 4\sqrt{\lambda}(1 - x^2 + y^2)(1 - 2x^2 - y^2)\mathcal{N}_4 \\
& + 2\lambda(3 - 4x^2 - 5y^2)\ln(y) + 8\lambda(1 - 2x^2 - y^2)\ln\left(\frac{xy}{\lambda}\right) \\
& + (3 - x^2 + y^2)(1 + 4x^2 - y^2)\sqrt{\lambda}\ln(w_1) \\
& + 8\lambda(1 - 2x^2 - y^2) \ln\left[\frac{(1+x)^2 - y^2}{x}\right] \\
& + 4 \left[ (1 - x^2)(5 - 4x^2 + 4x^4) \right. \\
& \quad \left. - (1 - x^2 + 2x^4)y^2 - 2(2 - x^2)y^4 \right] \ln\left(\frac{1-x}{y}\right) \\
& - \left[ (1-x)^2 - y^2 \right] \\
& \left. \times \left[ 15 + 2x - 5x^2 - 12x^3 + 2x^4 - (12 + 2x + 7x^2)y^2 - 3y^4 \right] \right\}
\end{aligned} \tag{23}$$

$$\begin{aligned}
\tilde{\Gamma}_{I^P} = & \mathcal{A} + \frac{1}{\lambda} \left\{ 2 \left[ 7 + 15x^2 + 4x^4 - (11 + 8x^2)y^2 + 4y^4 \right] \mathcal{N}_0 \right. \\
& \left. + 4\sqrt{\lambda}(1 - x^2 + y^2)\mathcal{N}_4 \right\}
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{x^2} (1 - 3x^2 - y^2) \lambda \ln(y) + 8\lambda \ln\left(\frac{xy}{\lambda}\right) \\
& + \frac{\sqrt{\lambda}}{2x^2} [1 + 30x^2 + 21x^4 - 2(1 + 11x^2)y^2 + y^4] \ln(w_1) \\
& + 8\lambda \ln\left[\frac{(1+x)^2 - y^2}{x}\right] \\
& + 2 \left[ (1-x^2)(21+5x^2) - (11-15x^2)y^2 - 4y^4 \right] \ln\left(\frac{1-x}{y}\right) \\
& - 2 \left[ (1-x)^2 - y^2 \right] (12 - 7x + 12x^2 - 9y^2) \Big\} \tag{24}
\end{aligned}$$

As mentioned before the contribution denoted by  $\mathcal{A}$  is that part of the finite remainder of the Born term type one-loop contribution plus the soft gluon contribution which contains dilogs and products or squares of logs . It is given by

$$\begin{aligned}
\mathcal{A} = & \frac{2}{\sqrt{\lambda}} (1 - x^2 + y^2) \Big\{ - 4 \text{Li}_2(1 - w_1) + 4 \text{Li}_2(1 - w_2) - 4 \text{Li}_2(1 - w_3) \\
& - \ln(w_1) \ln\left(\frac{\lambda^2 w_3}{x^2 y^3}\right) - \frac{1}{2} \ln^2(w_1) + \ln\left[\frac{1}{2} (1 - x^2 + y^2 + \sqrt{\lambda})\right] \ln(w_2 w_3) \Big\}. \tag{25}
\end{aligned}$$

The functions  $\mathcal{N}_{0,\dots,4}$  are defined by

$$\begin{aligned}
\mathcal{N}_0 &= \text{Li}_2\left(\frac{x}{\eta}\right) + \text{Li}_2(x\eta) - 2 \text{Li}_2(x) \\
\mathcal{N}_1 &= \text{Li}_2(x\eta) - \text{Li}_2\left(\frac{x}{\eta}\right) + 2 \ln(1 - \eta x) \ln\left[\frac{(\eta+1)x}{1+x}\right] + \ln\left(\frac{\eta}{\eta-x}\right) \ln\left[\frac{x^2(\eta-1)^2}{\eta(\eta-x)}\right] \\
\mathcal{N}_2 &= \text{Li}_2\left[\frac{(\eta-1)x}{\eta-x}\right] + \text{Li}_2\left[\frac{(\eta-1)x}{1-x}\right] - \frac{1}{2} \ln^2(1-x) + \ln\left(\frac{\eta}{\eta-x}\right) \ln\left[\frac{(\eta-1)x}{\eta-x}\right] \\
& + \ln(1-x) \ln\left(\frac{1-x}{\eta-x}\right) + \ln(1-\eta x) \ln\left[\frac{(\eta+1)x}{1+x}\right] \\
\mathcal{N}_3 &= \text{Li}_2\left(\frac{1-\eta x}{1+x}\right) - \text{Li}_2\left[\frac{\eta-x}{\eta(1+x)}\right] - \frac{1}{2} \ln\left(\frac{\eta}{\eta-x}\right) \ln\left[\frac{\eta(\eta-1)^2(1+x)^2}{(\eta+1)^2(\eta-x)}\right] \\
\mathcal{N}_4 &= 4 \text{Li}_2\left(\frac{\eta\sqrt{\lambda}}{\eta-x}\right) - 2 \text{Li}_2\left[\frac{(\eta-1)x}{1-x}\right] - 2 \text{Li}_2\left[\frac{(\eta-1)x}{\eta-x}\right] + \text{Li}_2\left(\frac{x}{\eta}\right) - \text{Li}_2(x\eta)
\end{aligned}$$

$$- \ln^2(1-x) + \ln\left(\frac{\eta}{\eta-x}\right) \ln\left[\frac{\eta(\eta+1)^2}{\eta-x}\right] + 2 \ln(1-\eta x) \ln\left[\frac{(1-x)(\eta+1)}{\eta-x}\right],$$

where we use the abbreviations

$$\begin{aligned} w_1 &= \frac{1-x^2+y^2-\sqrt{\lambda}}{1-x^2+y^2+\sqrt{\lambda}}, & w_2 &= \frac{1+x^2-y^2-\sqrt{\lambda}}{1+x^2-y^2+\sqrt{\lambda}}, & w_3 &= \frac{1-x^2-y^2-\sqrt{\lambda}}{1-x^2-y^2+\sqrt{\lambda}} \\ \eta &= \frac{1+x^2-y^2+\sqrt{\lambda}}{2x} \end{aligned} \quad (26)$$

## 5 Nonperturbative contributions

When one uses the operator product expansion in HQET one can determine the nonperturbative corrections to the leading partonic  $b \rightarrow c$  rate. The nonperturbative corrections set in at  $O(1/m_b^2)$  and arise from the kinetic energy and the spin dependent piece of the heavy quark in the heavy baryon [13]. The strength of the kinetic and the spin dependent piece are parametrized by the expectation values of the relevant operators in the  $\Lambda_b$  system and are denoted by  $K_b$  and  $\epsilon_b$ , respectively. We have completely recalculated the nonperturbative contributions to the seven partial rates and have found some errors in the calculation of [14] which will be corrected in an Erratum to [14]. One has

$$\begin{aligned} S : \quad a_S^K &= -1, & a_S^\epsilon &= 0 \\ U : \quad a_U^K &= -\left(1 - \frac{8}{3} \frac{1}{1-x^2+y^2}\right), & a_U^\epsilon &= 0 \\ L : \quad a_L^K &= -\left(1 + \frac{16}{3} \frac{x^2}{(1-y^2)^2 - x^2(1+y^2)}\right), & a_L^\epsilon &= 0, \\ U+L : \quad a_{U+L}^K &= -1, & a_{U+L}^\epsilon &= 0 \\ S^P : \quad a_{S^P}^K &= -(1 + \frac{8}{3} \frac{x^2}{\lambda}), & a_{S^P}^\epsilon &= 1 \\ U^P : \quad a_{U^P}^K &= -(1 + \frac{8}{3} \frac{x^2}{\lambda}) & a_{U^P}^\epsilon &= 1 \\ L^P : \quad a_{L^P}^K &= -(1 + \frac{8}{3} \frac{x^2}{\lambda}) & a_{L^P}^\epsilon &= 1 \\ (U+L)^P : \quad a_{(U+L)^P}^K &= -(1 + \frac{8}{3} \frac{x^2}{\lambda}) & a_{(U+L)^P}^\epsilon &= 1 \\ I^P : \quad a_{I^P}^K &= \frac{2}{3}(1 - 4 \frac{x^2}{\lambda}) & a_{I^P}^\epsilon &= 1 \end{aligned} \quad (27)$$

The nonperturbative contributions for  $(U+L)$  and  $(U+L)^P$  can be compared to the corresponding  $q^2$ -distributions in semi-leptonic  $b$ -decays written down in [15]. We find

agreement.

For our numerical evaluation we use  $K_b = 0.013$  for the mean kinetic energy of the heavy quark in the  $\Lambda_b$  as in [14]. An estimate of the spin-dependent parameter has been given in [16] with the result  $\epsilon_b = -\frac{2}{3}K_b$ , based on an assumption that the contribution of terms arising from double insertions of the chromomagnetic operator can be neglected. A zero recoil sum rule analysis gives the constraint  $\epsilon_b \leq -\frac{2}{3}K_b$  [17] which puts the estimate of [16] at the upper bound of the constraint. We use the value of [16] keeping in mind that the numerical value of  $\epsilon_b$  could be reduced in more realistic calculations.

## 6 Numerical results

Using  $m_b = 4.85$  GeV,  $m_c = 1.45$  GeV,  $m_{D_s} = 1968.5$  MeV,  $m_{D_s^*} = 2112.4$  MeV and  $\alpha_s(m_b) = 0.2$  we obtain for  $b \rightarrow c$

$$\begin{aligned}
\hat{\Gamma}_S &= (1 - 0.0964 - 0.0130 + 0), \\
\hat{\Gamma}_U &= 0.3541 (1 - 0.1079 + 0.0255 + 0), \\
\hat{\Gamma}_L &= 0.6459 (1 - 0.1103 - 0.0341 + 0), \\
\hat{\Gamma}_{U+L} &= (1 - 0.1095 - 0.0130 + 0), \\
\hat{\Gamma}_{S^P} &= 0.9884 (1 - 0.1027 - 0.0245 - 0.0087), \\
\hat{\Gamma}_{U^P} &= -0.2646 (1 - 0.0616 - 0.0276 - 0.0087), \\
\hat{\Gamma}_{L^P} &= 0.6351 (1 - 0.1043 - 0.0276 - 0.0087), \\
\hat{\Gamma}_{(U+L)^P} &= 0.3705 (1 - 0.1348 - 0.0276 - 0.0087), \\
\hat{\Gamma}_{I^P} &= -0.2148 (1 - 0.0876 + 0.0059 - 0.0087).
\end{aligned} \tag{28}$$

The four entries in the round brackets correspond to the Born term contribution, the  $O(\alpha_s)$  corrections, and the nonperturbative kinetic and spin-dependent corrections in that order, as specified in Eq. (15).

The reduction of the partial rates from the radiative corrections scatter around 10%, where the reduction is largest for  $\hat{\Gamma}_{(U+L)^P}$  (−13.5%) and smallest for  $\hat{\Gamma}_{U^P}$  (−6.2%). When normalized to the total rate, as is appropriate for density matrix elements, the corresponding density matrix elements are reduced by 2.84% and increased by 5.38% in magnitude by the radiative corrections, respectively.

The nonperturbative corrections range from  $-0.9\%$  for the spin-dependent corrections to a maximal  $-3.4\%$  for the kinetic energy correction to  $\hat{\Gamma}_L$ . The nonperturbative corrections are all negative except for the kinetic energy correction to  $\hat{\Gamma}_U$  and  $\hat{\Gamma}_{IP}$ .

As specified in Eqs. (5) and (6), the asymmetry parameters  $\alpha_P(D_s, D_s^*)$  can be measured in the semi-inclusive decays of a polarized  $\Lambda_b$  into the two decay channels. For the pseudoscalar case the Born term level asymmetry  $\alpha_P(D_s) = 0.99$  is quite close to its maximal attainable value of 1 which would be achieved for  $y = 0$ . The Born term value is only slightly reduced to  $\alpha_P(D_s) = 0.97$  by the radiative and nonperturbative corrections. For the vector case the asymmetry parameter is smaller. At Born term level one has  $\alpha_P(D_s^*) = 0.37$  which is reduced to  $\alpha_P(D_s^*) = 0.35$  including the radiative and nonperturbative corrections. As outlined in Sec. 2 the transverse/longitudinal composition of the  $D_s^*$  can be measured by the  $\cos\theta$ -dependence in the angular decay distribution of its decay products. At the Born term level the transverse/longitudinal composition is given by  $\hat{\Gamma}_U/\hat{\Gamma}_L = 0.55$ . This ratio is shifted upward by the insignificant amount of  $0.3\%$  through the radiative corrections. Adding all corrections one finds a  $7.3\%$  enhancement in the  $U/L$  ratio.

Next we turn to our numerical results for the Cabibbo-suppressed semi-inclusive decays  $\Lambda_b \rightarrow X_u + (D_s, D_s^*)$  induced by the  $b \rightarrow u$  transitions. Compared to the above Cabibbo-enhanced semi-inclusive decays they are down by a factor  $(V_{ub}/V_{cb})^2 \approx 10^{-2}$ , which is only slightly compensated for by a kinematical enhancement factor of  $\approx 1.5$ . Setting  $m_u = 0$ , i.e.  $y = 0$ , one has

$$\begin{aligned}
\hat{\Gamma}_S &= (1 - 0.1694 - 0.0130 + 0), \\
\hat{\Gamma}_U &= 0.2750 (1 - 0.1150 + 0.0285 + 0), \\
\hat{\Gamma}_L &= 0.7250 (1 - 0.1777 - 0.0267 + 0), \\
\hat{\Gamma}_{U+L} &= (1 - 0.1605 - 0.0130 + 0), \\
\hat{\Gamma}_{SP} &= (1 - 0.1745 - 0.0212 - 0.0087), \\
\hat{\Gamma}_{UP} &= -0.2750 (1 - 0.1275 - 0.0212 - 0.0087), \\
\hat{\Gamma}_{LP} &= 0.7250 (1 - 0.1800 - 0.0212 - 0.0087), \\
\hat{\Gamma}_{(U+L)P} &= 0.4500 (1 - 0.2121 - 0.0212 - 0.0087), \\
\hat{\Gamma}_{IP} &= -0.2233 (1 - 0.1506 + 0.0005 - 0.0087).
\end{aligned} \tag{29}$$

In the  $b \rightarrow u$  case the radiative corrections and their spread are larger than in the  $b \rightarrow c$  case. The reduction of the partial rates from the radiative corrections now scatter around  $-17\%$ , where the reduction is largest for  $\hat{\Gamma}_{(U+L)^P}$  ( $-21.2\%$ ) and smallest for  $\hat{\Gamma}_U$  ( $-11.5\%$ ). When normalized to the total rate, the corresponding density matrix elements are reduced by  $6.1\%$  and increased by  $5.4\%$  in magnitude by the radiative corrections, respectively. The dominance of the longitudinal rate is now more pronounced. At the Born term level one finds  $\Gamma_U/\Gamma_L = 2x^2 = 0.38$ . The ratio  $\Gamma_U/\Gamma_L$  is shifted upward by  $7.6\%$  by the radiative corrections. Adding up all corrections one finds a  $14.8\%$  upward shift for this ratio. For the asymmetry parameter one obtains  $\alpha_P(D_s) = 0.984$  including all corrections which shifts the uncorrected result  $\alpha_P(D_s) = 1$  downward by  $1.6\%$ . For the asymmetry parameter  $\alpha_P(D_s^*)$  one obtains  $\alpha_P(D_s^*) = 0.42$  which is lower than the uncorrected result of  $\alpha_P(D_s^*) = 0.45$  by  $7.3\%$ . Let us mention that our  $O(\alpha_s)$  results on  $\Gamma_{U+L}$  and  $\Gamma_S$  numerically agree with the results of [1] for both the  $b \rightarrow c$  and  $b \rightarrow u$  transitions.

As a last point we want to discuss the semi-inclusive decays  $\Lambda_b \rightarrow X_c + (\pi^-, \rho^-)$  which have not been discussed so far. They are also induced by the diagrams Fig. 2 when the  $c \rightarrow s$  transition in the upper leg is replaced by a  $u \rightarrow d$  transition. Using  $f_{\pi^-} = 132$  MeV,  $f_{\rho^-} = 216$  MeV and  $V_{ud} = 0.975$  one finds the Born term branching fractions  $\text{BR}_{b \rightarrow \pi^- + c} \cong 1.6\%$  and  $\text{BR}_{b \rightarrow \rho^- + c} \cong 4.6\%$ . In the latter case the rate is dominated by the longitudinal contribution since  $q^2 = m_\rho^2$  is not far from  $q^2 = 0$  where the rate would be entirely longitudinal. In fact one finds  $\Gamma_U/\Gamma_L = 0.067$ . It is important to note that the diagrams Fig. 2 are not the only mechanisms that contribute to the semi-inclusive decays  $\Lambda_b \rightarrow X_c + (\pi^-, \rho^-)$ . Additional  $\pi^-$  and  $\rho^-$  mesons can also be produced by fragmentation of the  $c$ -quark at the lower leg.<sup>3</sup> As concerns the  $\rho^-$  mesons resulting from the fragmentation process they would not be polarized along their direction of flight. This lack of polarization as compared to the strong polarization of the  $\rho$  mesons from the weak vertex could possibly be used to separate  $\rho^-$  mesons coming from the two respective sources.

## 7 Summary and conclusions

We have calculated the perturbative  $O(\alpha_s)$  and the nonperturbative  $O(1/m_b^2)$  corrections to the seven structure functions that can be measured in the semi-inclusive decay of a polarized  $\Lambda_b$  in the process  $\Lambda_b(\uparrow) \rightarrow X_c + (D_s^-, D_s^{*-})$ . We have used the factorization

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<sup>3</sup>As concerns the semi-inclusive decays  $\Lambda_b \rightarrow X_c + (D_s^-, D_s^{*-})$  the possibility of producing extra  $D_s^-$  and  $D_s^{*-}$  mesons through fragmentation of the  $c$ -quark is ruled out for kinematic reasons.

hypothesis to factorize the semi-inclusive decay into a current-induced  $\Lambda_b \rightarrow X_c$  transition and a current-induced vacuum one-meson transition. The dominant contribution to the current-induced  $\Lambda_b \rightarrow X_c$  transition is given by the leading order HQET transition  $b \rightarrow c$ . Thus the semi-inclusive decays of a polarized  $\Lambda_b$  offer the unique opportunity to measure seven of the nine structure functions that describe the current-induced free quark transition  $b \rightarrow c$ .

We emphasize that there are also nonfactorizing  $O(\alpha_s)$  contributions which have not been included in our analysis. However, the nonfactorizing  $O(\alpha_s)$  contributions are colour suppressed and are thus expected to be small.

We find that the perturbative corrections are always negative. The nonperturbative corrections are negative in most of the cases. The net effect of the corrections to the structure functions can become as large as  $-20\%$  for the  $b \rightarrow c$  transitions and can exceed  $-20\%$  for the  $b \rightarrow u$  transitions. When normalized to the total rate, as is appropriate for density matrix elements accessible to experimental measurement, the corrections become smaller but can still amount to  $\approx \pm 5\%$ .

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